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3-Share Threshold Implementation of AES S-box without Fresh Randomness

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Overview

Methodology

Threshold implementation
(Nicova et al., ICICS2006)

Changing of the guards
(Daemen, CHES2017)

Generalized
Changing of the guards
(This work)

Implementation

Difficulty in realizing
3-share + Uniform TI for
AES and Keccak for 10+ years

3-Share + Uniform Keccak S-box
(Daemen, CHES2017)

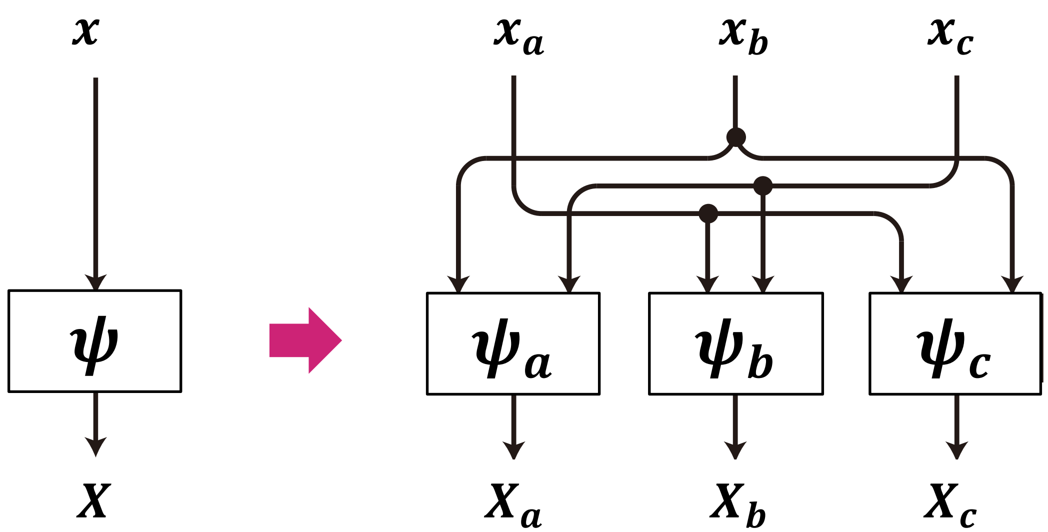
4-Share + Uniform AES S-box
(Wegener & Moradi, COSADE2018)

3-Share + Uniform AES S-box
(This work)

TI: Threshold Implementation

- Implement crypto while keeping shared representation of intermediate variables

Input share (x_a, x_b, x_c) :
$x_a \oplus x_b \oplus x_c = x$



Output share (X_a, X_b, X_c) :
$X_a \oplus X_b \oplus X_c = X$

Sharing $\{\psi_a, \psi_b, \psi_c\}$ maps a share to another share

Correctness:
$\{\psi_a, \psi_b, \psi_c\}$ gives the correct result

Non-completeness:
Each map uses only a proper subset

Uniformity

- **Uniformity about shares**
  - For each (raw) value, all the possible shares should appear equally
  - Necessary for security against statistical attack

- **Uniformity about sharing**
  - The sharing preserves the uniformity about shares:

    Input share is uniform
    $\Longrightarrow$ output share is uniform

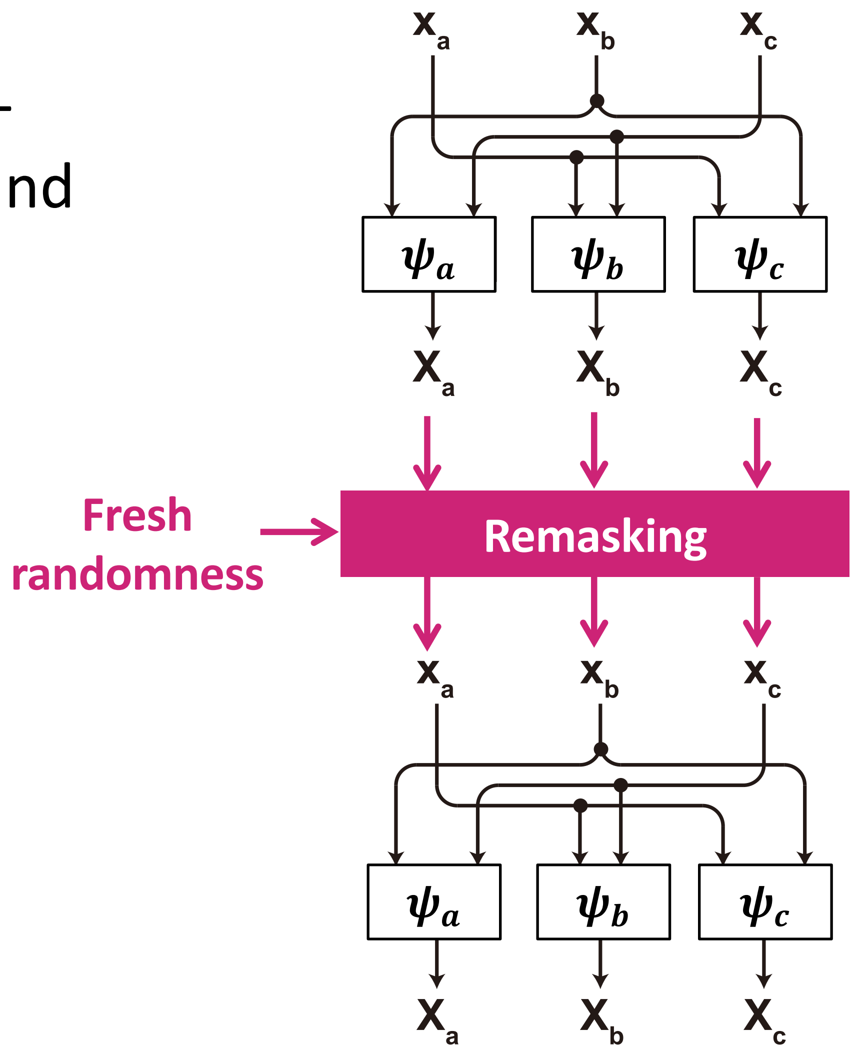
Example:
3-share of 1-bit variable

Raw value	Share	Prob.
0	(0,0,0)	1/16
0	(0,1,1)	1/16
0	(1,0,1)	1/16
0	(1,1,0)	1/16
1	(0,0,1)	3/16
1	(0,1,0)	3/16
1	(1,0,0)	3/16
1	(1,1,1)	3/16

Uniformity is difficult to satisfy

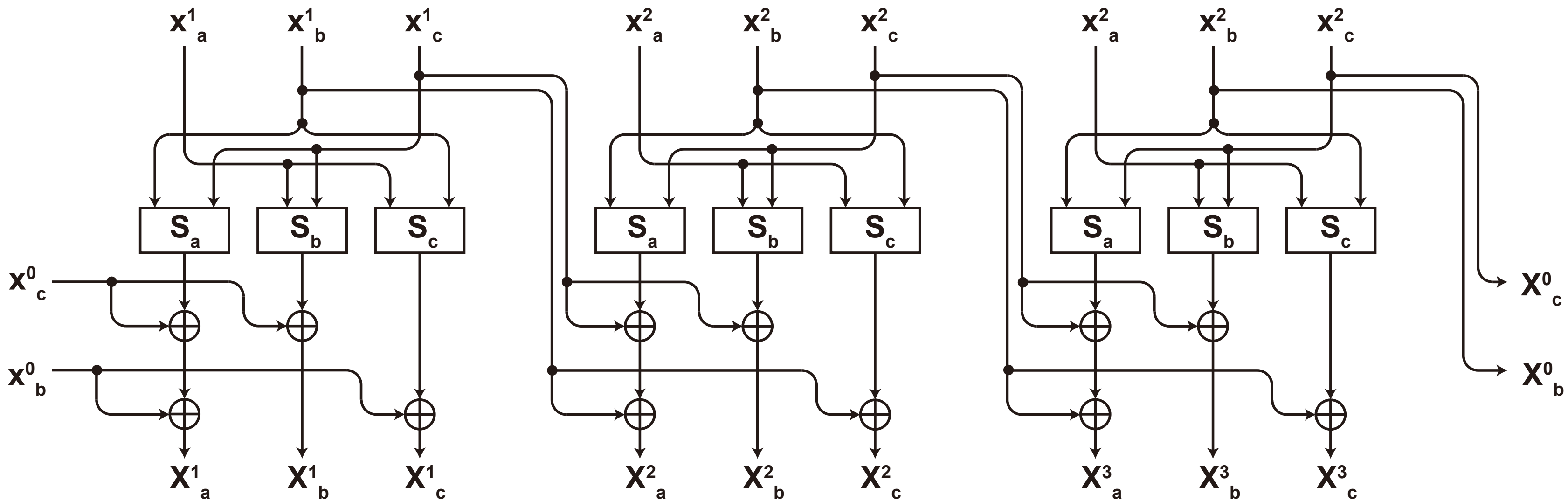
- There had been no 3-share + uniform sharing for Keccak and AES S-boxes until 2017

- If no uniformity, we should add fresh randomness to make the output share uniform again
  - 1—10 Kbits/AES
  - 10—50 bits/cycle



CotG: Changing of the Guards (Daemen, CHES2017)

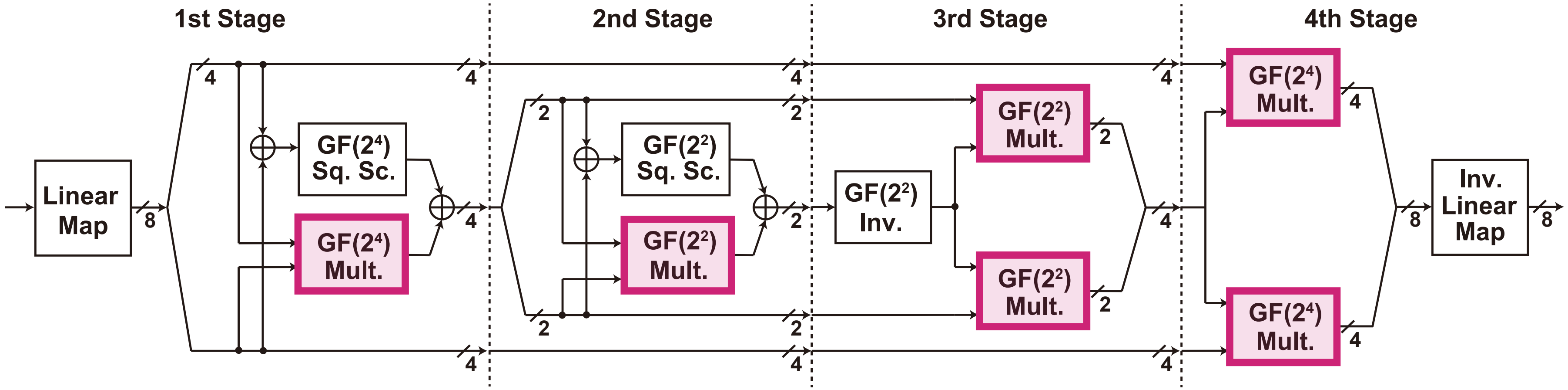
- Using a neighboring input share for (pseudo) remasking
- **Applicable to bijective mapping**
  - Succeeded in making 3-share + uniform Keccak S-box



Why we can't use CotG for 3-share AES S-box

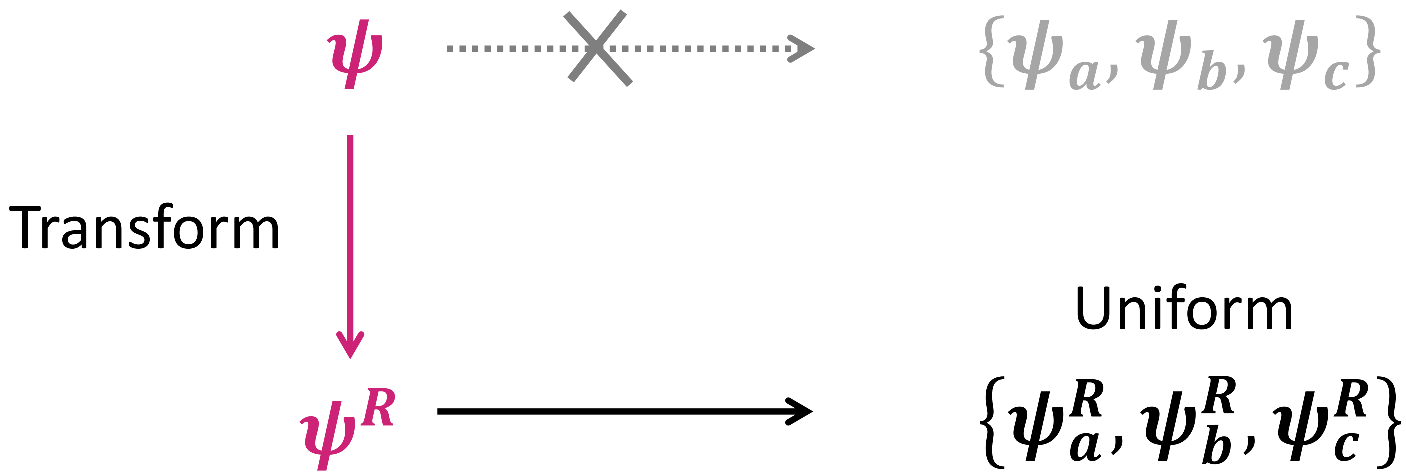
- We need to decompose S-box to reduce the number of shares, and we get **multiplications that are not bijective**

Canright's S-box implementation



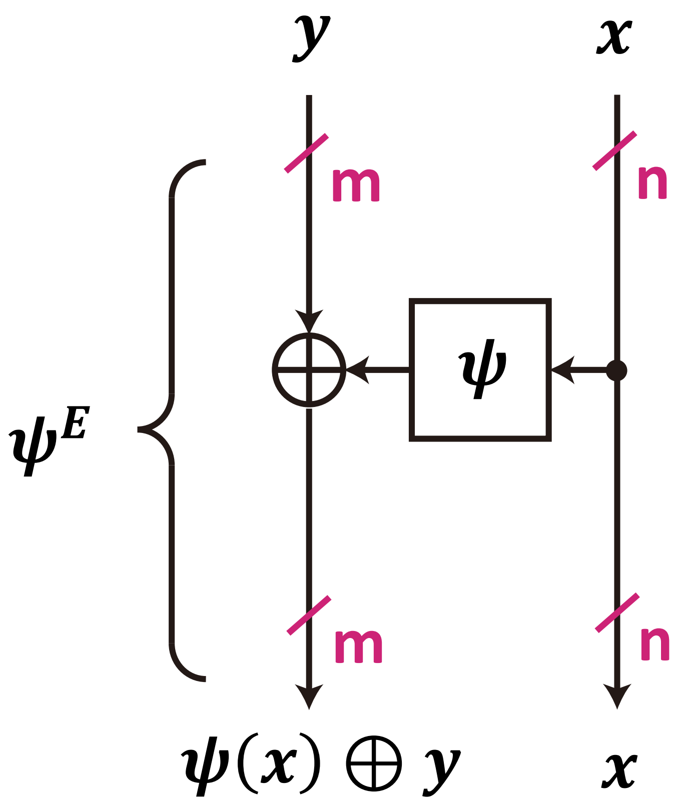
Basic idea toward generalization

- Transform the target mapping $\boldsymbol{\psi}$ into an **equivalent** mapping $\boldsymbol{\psi^R}$ that has a uniform sharing



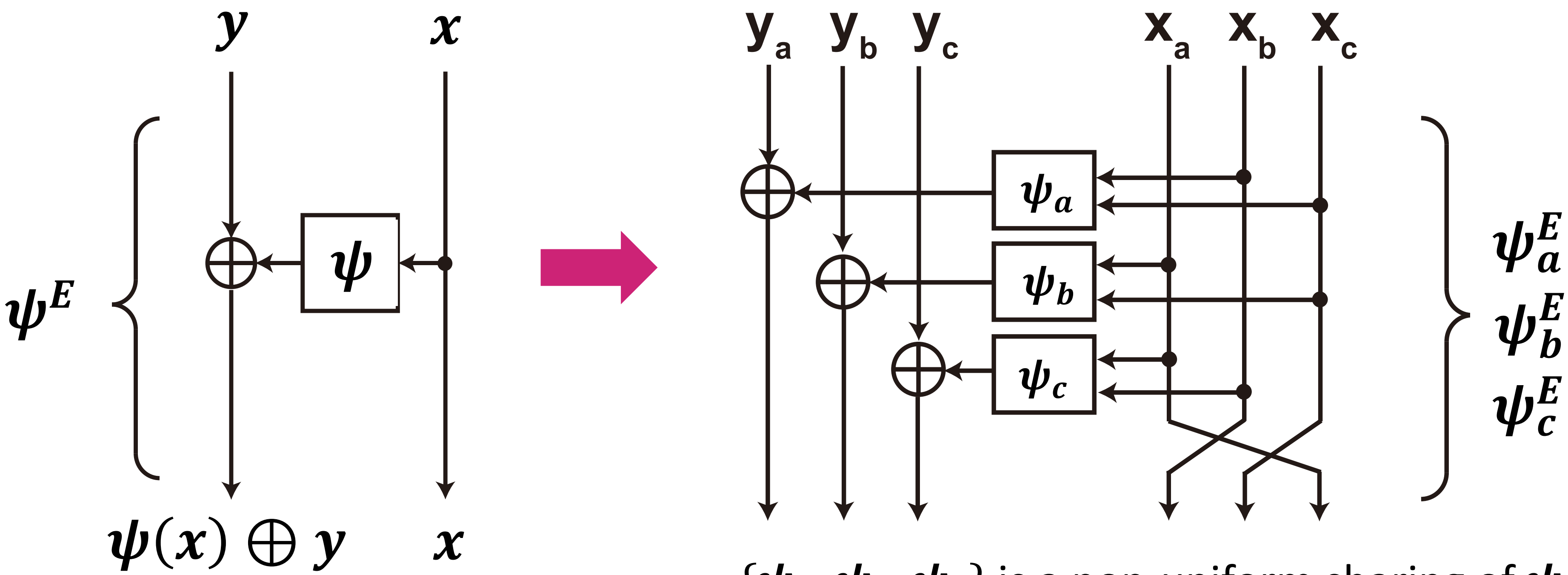
Expansion

- Transforming the target $\boldsymbol{\psi}$ into a bijective mapping $\boldsymbol{\psi}^{\boldsymbol{E}}$ using **the (unbalanced) Feistel network**



Expansion cont.

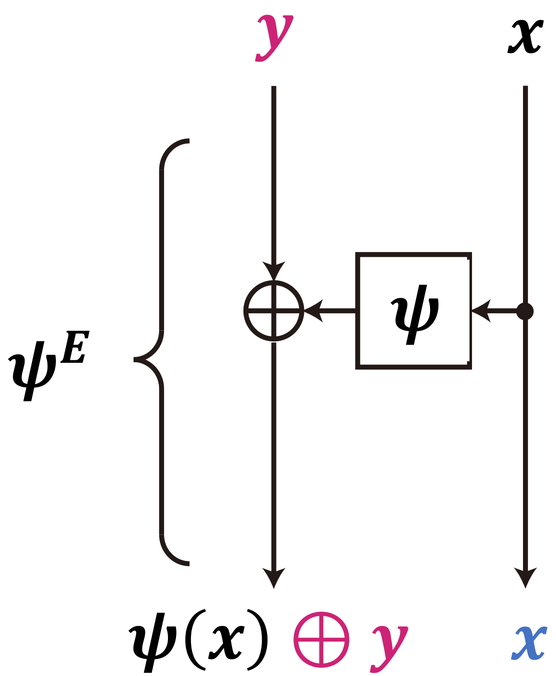
- ψ^E **always has a uniform sharing** $\{\psi_a^E, \psi_a^E, \psi_a^E\}$
  - ∵ The sharing is bijective because the Feistel structure is preserved
  - ∵ A sharing is bijective $\Longrightarrow$ the sharing is uniform



$\{\psi_a, \psi_b, \psi_c\}$ is a non-uniform sharing of ψ

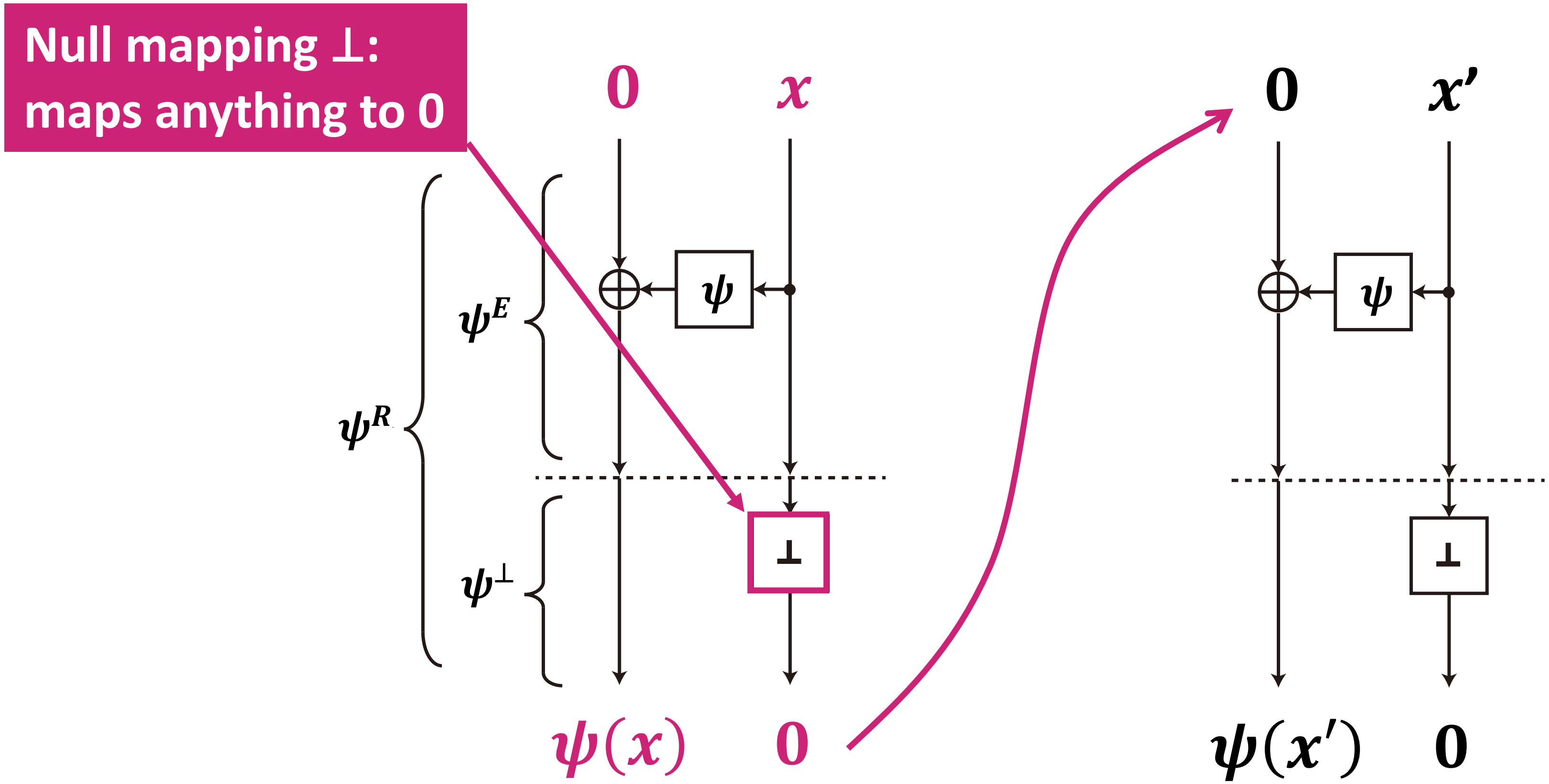
Expansion is not enough

- Feeding $\boldsymbol{\psi}^{\boldsymbol{E}}(\boldsymbol{x})$ to CotG does not make a lot of sense since it outputs $\boldsymbol{\psi}(\boldsymbol{x}) \oplus \boldsymbol{y}$ instead of $\boldsymbol{\psi}(\boldsymbol{x})$
- **$\boldsymbol{y}$ should be 0 and we need to get it from somewhere**



Restriction

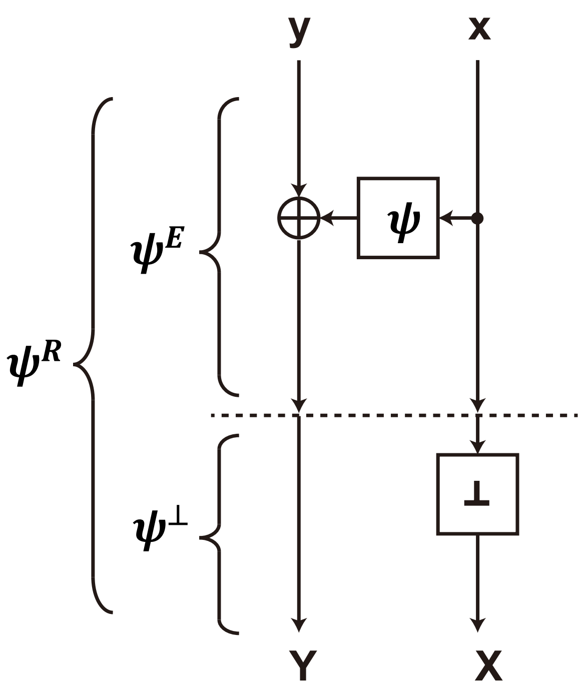
- **Converting the unnecessary output to zero**
- Feeding it to a neighboring mapping as a zero input

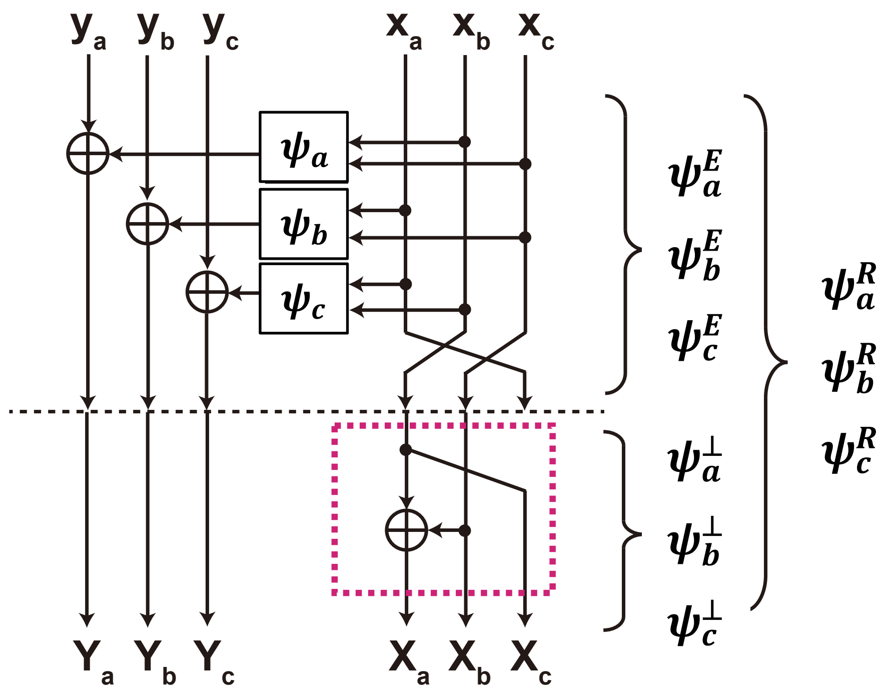


Restriction cont.

- The null mapping ⊥ has a uniform sharing
  - $\{x_a, x_b, x_c\} \mapsto \{x_b \oplus x_c, x_b, x_c,\}$

Converting unnecessary share to another one representing 0



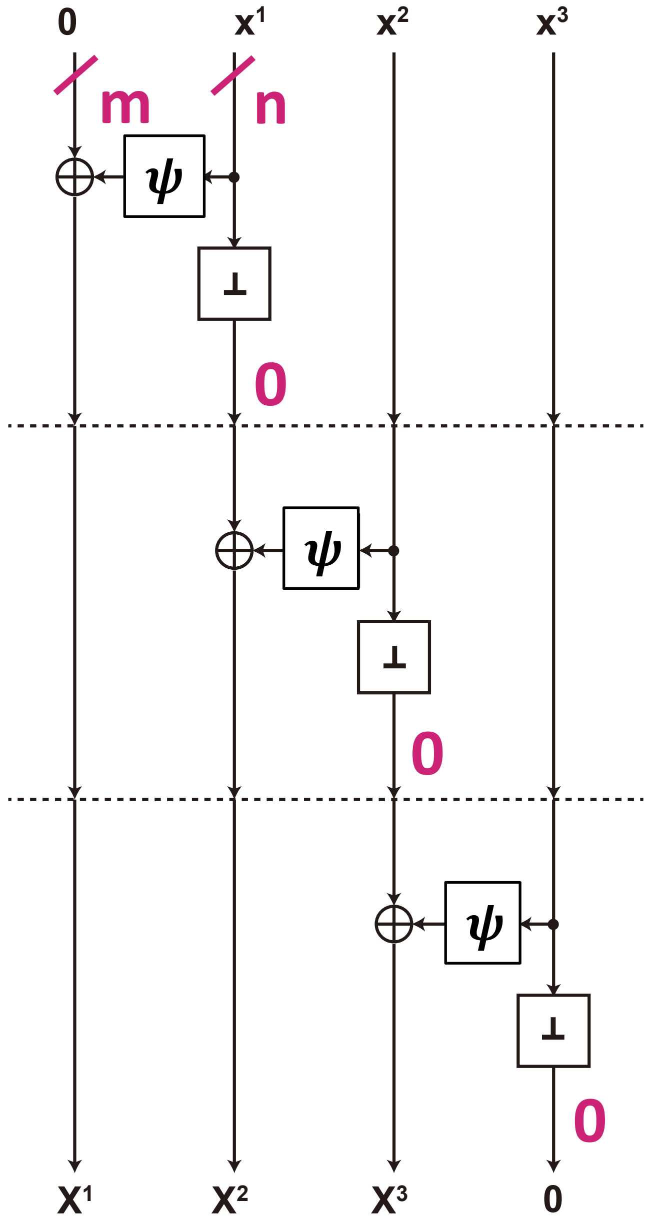


Chaining

- For a target map having the same input and output sizes ($\boldsymbol{m} = \boldsymbol{n}$), we can easily chain zero outputs and inputs

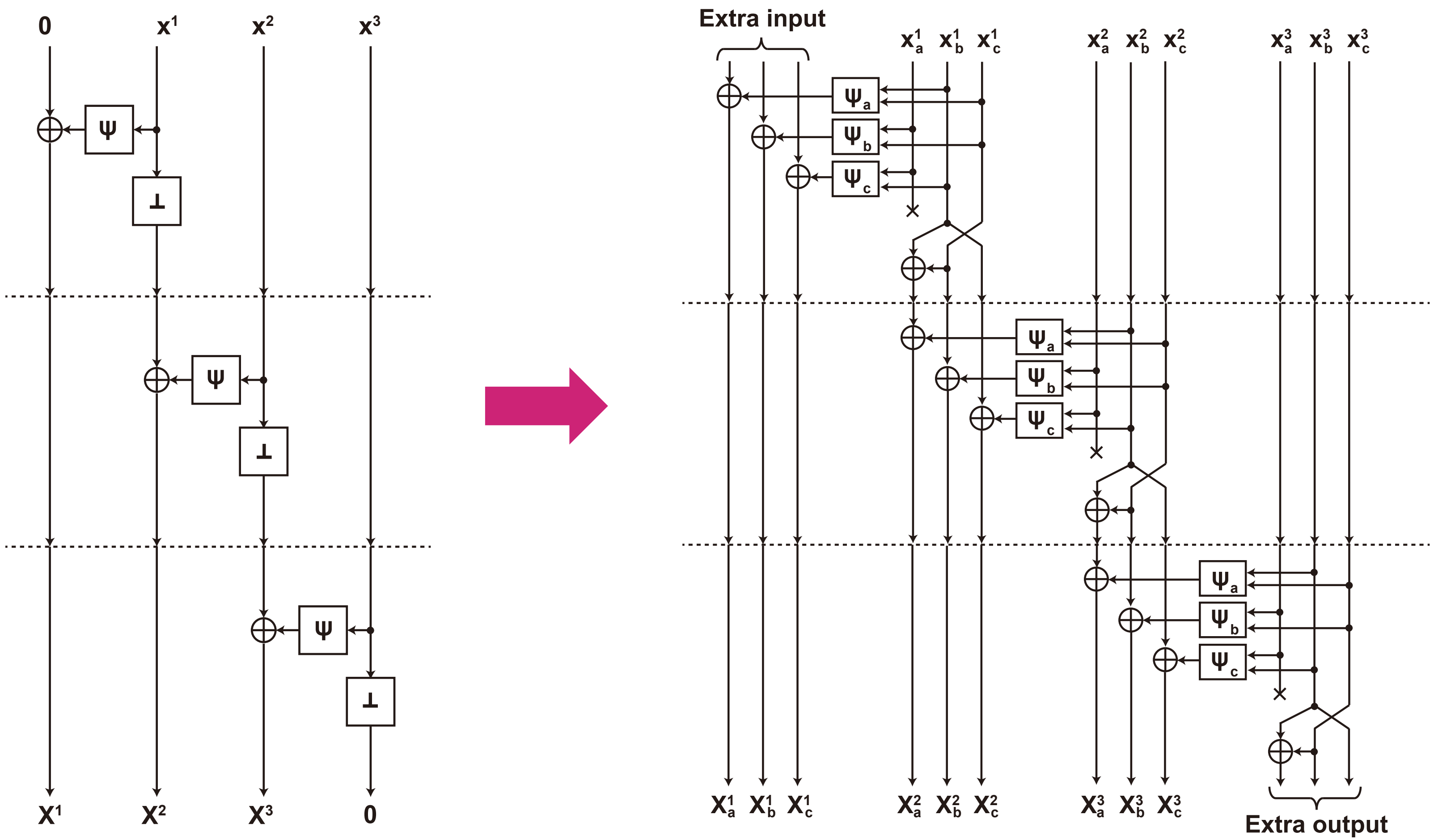
- The right figure shows 3-parallel mapping given by

$$(\mathbf{0}, \boldsymbol{x}^1, \boldsymbol{x}^2, \boldsymbol{x}^3) \mapsto (\boldsymbol{\psi}(\boldsymbol{x}^1), \boldsymbol{\psi}(\boldsymbol{x}^2), \boldsymbol{\psi}(\boldsymbol{x}^3), \mathbf{0})$$



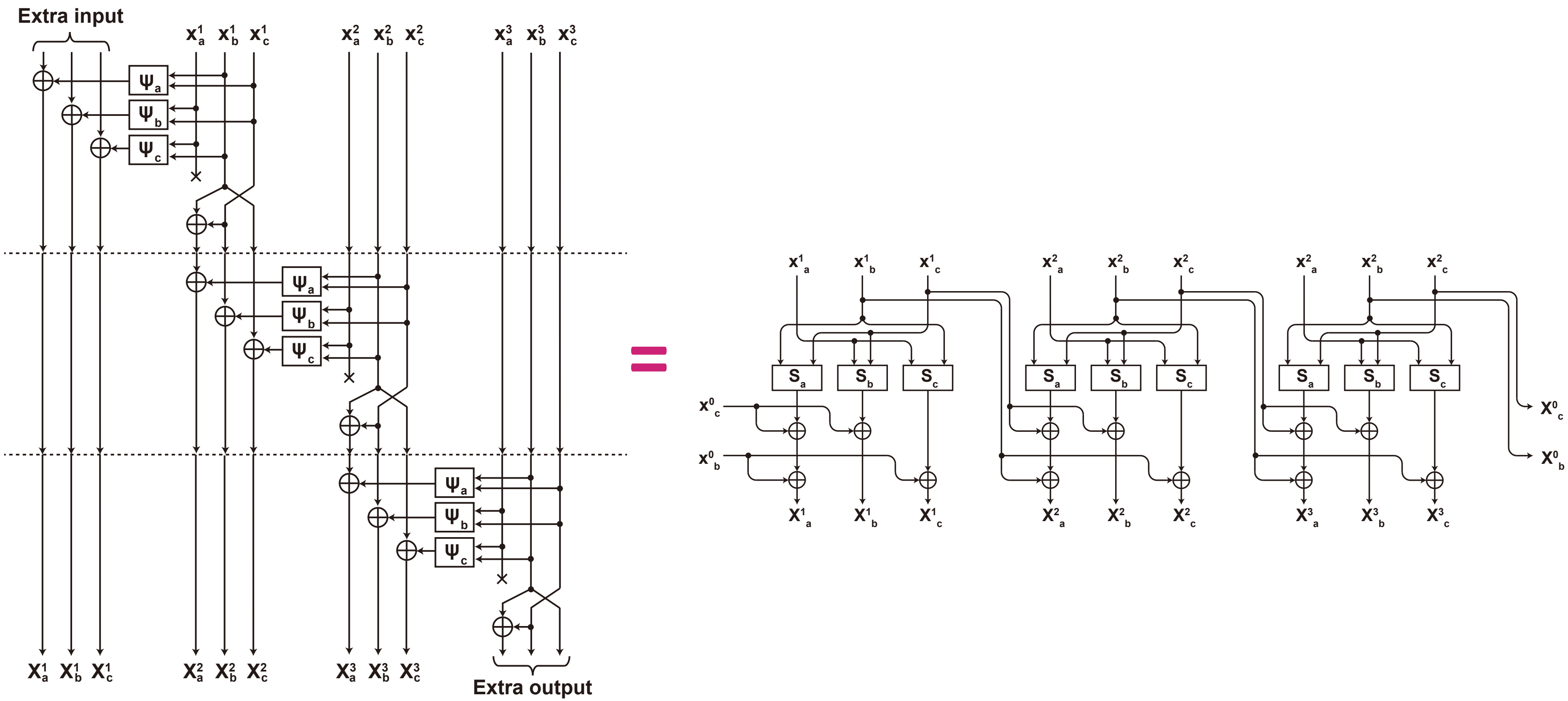
Chaining cont.

- By substituting each $\boldsymbol{\psi}^{\boldsymbol{R}}$ with its sharing, we get a uniform sharing of a layer of parallel $\boldsymbol{\psi}^{\boldsymbol{R}}$s



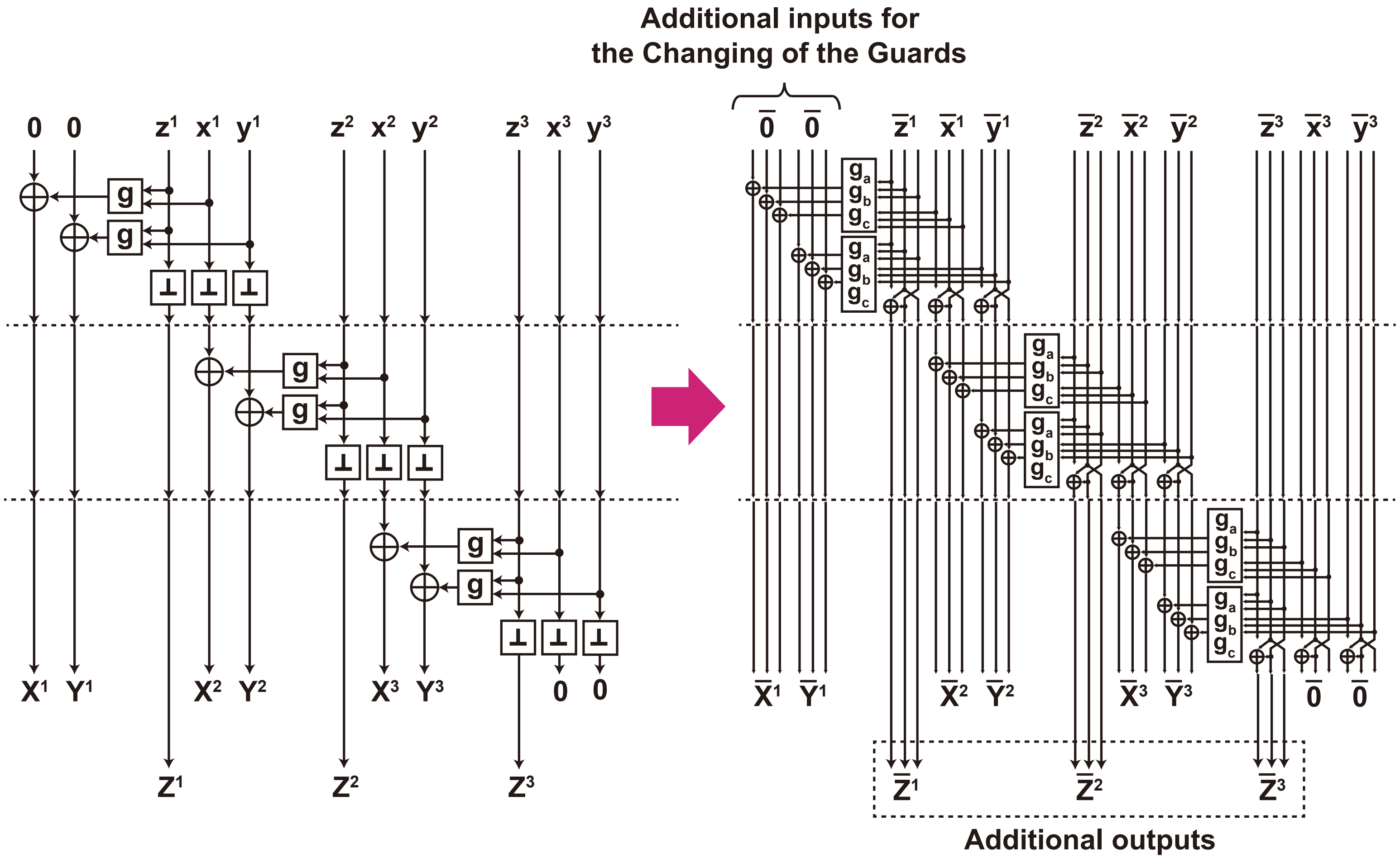
Why it is a generalization of CotG

- **This sharing is the same as Daemen's CotG**
- Now we can also support non-bijective mapping



A map with different input/output sizes

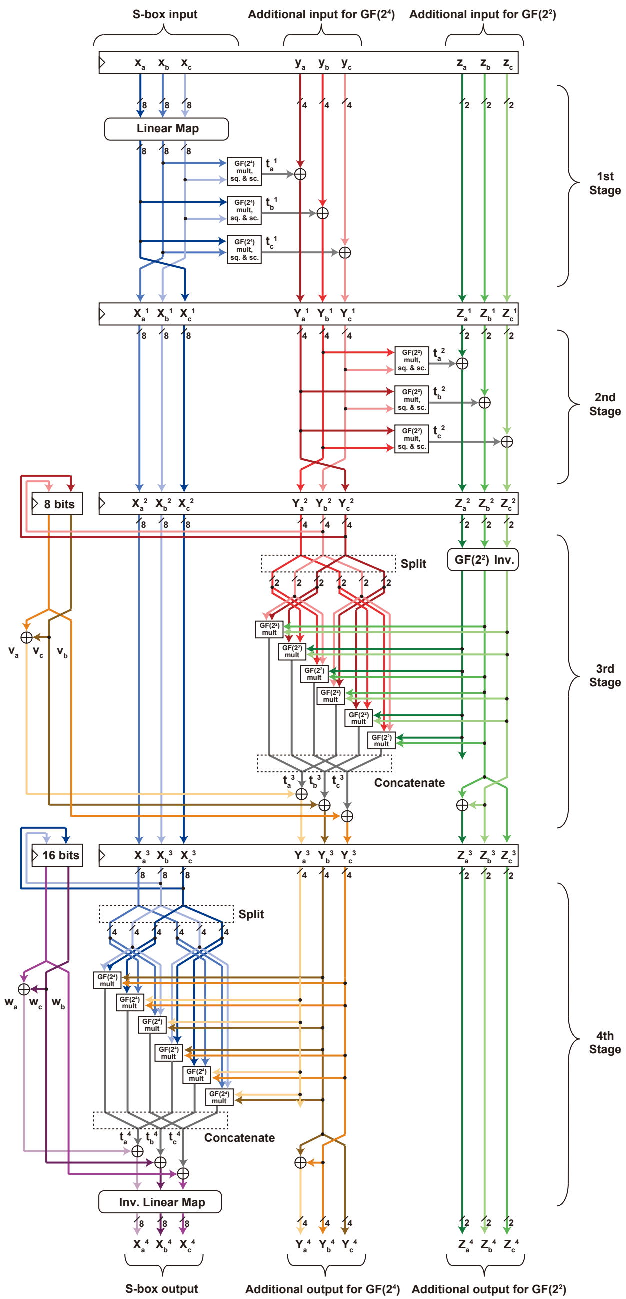
- Input is larger: we get additional zero outputs that we can use later
- Output is larger: we need additional zero inputs



Application to AES S-box

- 4-stage Canright's S-box is expanded to make all the stages uniform
  - + **6-bit** additional input
  - + **6-bit** additional output

- Register overhead ≒ Initial randomness:
  - **6 bits** * 3 shares *16 S-boxes = 288 bits + some more



Conclusion

- A generalization of the Changing of the Guards that supports non-bijective targets

- The first 3-share and uniform threshold implementation of the AES S-box